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ABSTRACT

This paper is a reflection of the preparation of the Psychology of Mathematics Education Panel (PME26) that discuss the issue of "learning from learners." What it implies to be a learner is formulated in order to reflect upon the way teachers and/or researchers learn from learners. The idea of conceptualizing the notion of "learning from learners" contains a rethinking of learning in terms of framing it by context, communication, and socially abandoning traditional views that focus upon isolated and individual subjects in confrontation with a cognitive or learning task. This implies the abandonment of the universalistic idea that learners are the same in all times and places. The paper reflects these notions in different ways. (KHR)

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LEARNING FROM LEARNERS

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INTRODUCTION – JOÃO FILIPE MATOS

When looking closely at everyday activity it is difficult to avoid the conclusion that learning is ubiquitous in ongoing activity. Nevertheless, societies came to forms of pedagogical practice that conform with the idea that social arrangements can be made up to provoke specific learning effects in some of the participants in a certain practice. Schooling is perhaps one of the most apparent examples of how societies create and maintain stable social practices. As soon as one enrols in school, at any level, one is positioned and labelled as a *learner*. This is not surprising but it strikes me the fact that as soon as one is positioned as a teacher, at any level, one tends to suspend or forget our condition of learners.

This paper is the reflection of the preparation of the PME26 Panel aiming to discuss the issue of 'learning from learners'. When addressing the topic of Learning from Learners, one of the key issues is what it implies to be a learner. Sinha (1999) frames this issue asking how the developing human is being constructed and positioned in particular and specific kinds of practices, in such a way that they become a learner of the kind required by the culture within which teaching-learning activities occur. This question is asked implicitly in most studies addressing mathematics learning. But within this paper it is formulated in order to reflect on the way we as teachers and/or researchers learn from learners— pupils, teachers, researchers.

The panellists invited to address this topic—Joop van Dormolen, Susie Groves and Rosetta Zan—accepted the challenge of producing a short narrative and a brief analysis of an episode that each one found relevant for their own learning as a teacher and/or as a researcher. They were also asked to comment on each other episode and combine their own analysis with the others'. The result was a joint paper putting together the reflection of the three.

From my point of view, the very idea of conceptualizing the notion of 'learning from learners' contains in itself a rethinking of learning in terms of framing it by context,

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communication and social practice abandoning traditional views which focus upon *isolated and individual subject* in confrontation with a cognitive or learning task. This implies, for example, the abandonment of the universalistic idea that learners, in all essentials, are the same in all times and places. These notions are reflected in different ways in the several parts of the paper.

EPISODE I: HOW KARIN DISTURBED HER TEACHER'S LINE OF REASONING - JOOP VAN DORMOLEN

The episode

The episode that I want to describe happened during a mathematics lesson in grade 7 that I observed. I was sitting together with a student teacher, let us call him John, in the back of the class, while the lesson was given by John's mentor, let us call him Wilbur. The episode took place in the end of September, four or five weeks after the start of the school year. Wilbur, the teacher, had a 15 to 20 year experience with teaching grades 7 to 12. Students liked him and trusted him as a teacher.

This was the first day that John had started his teaching practice period. It was decided that he should not right away start giving lessons, but first observe lessons given with his mentor. I was there as the John's university teacher. Wilbur and I had agreed that I would be in his class the first time that John was there and that after the lesson the three of us would sit together to analyze the lesson. This analysis in itself was meant to be a learning experience for John.

At the time of this story I had 20 years experience in the same kind of school as Wilbur and for about 6 years had been a teacher educator for secondary school mathematics at the university.

The students

The school was a typical Dutch school in which students would follow either a six years course that prepared for university studies or a five years course that prepared for higher non-university studies. Grade 7 is the first class of such schools. Decisions of which of the two courses the students would follow were not to be made before the end of the second year.

The subject matter

The lesson was about basic algebra rules: the commutative rules $a + b = b + a$ and $a \times b = b \times a$, and the distributive rule $a \times (b + c) = a \times b + a \times c$. During the weeks before the lesson the students had learned the concept of a variable and had had exercises with substitutions of numbers for variables and solving simple first degree equations with one variable.

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The event

Wilbur had explained the fact that it does not matter whether you add 3 to 2 or 2 to 3, you end up with the same result, and this not only for the numbers 2 and 3, but for any two numbers. We can express this with $a + b = b + a$ in which a and b stand for any number one can think of. This is not only for additions. Also in multiplication we have the same kind of rule: 3×2 means $2 + 2 + 2$ and 2×3 means $3 + 3$. These are two completely different multiplications, but their product is the same. So for any pair of numbers, hence we can say $a \times b = b \times a$.

Most of the time Wilbur was speaking and explaining with occasional questions to students. The class was attentive and seemed to be willing to learn.

Then Wilbur started with examples of the distributive rule. He had been working on this for some time, say 8 to 10 minutes, when one of the students, let us call her Karin, put up her hand. Wilbur, thinking that she wanted to comment on what he was discussing, asked her what she wanted to say.

She said: "Two time three is six because two times three is one plus one plus one ...*(here a slight pause)* plus one plus one plus one. And three times two is one plus one ...(*slight pause*) plus one plus one ... (*slight pause*) plus one plus one. In both cases you have the same number of one's.". While she spoke she moved her finger as if writing down in the air what she was saying.

In a friendly way Wilbur dismissed Karin's remark with something like: "Yes, you are right, but we saw that already, we are now doing something else." and went on with talking about the distributive rule.

What did I learn from Karin's remark?

Sitting in the back of the classroom I had not Wilbur's preoccupation of having to go on with the lesson, I could reflect on what Karin had said and I decided to make that one of the subject of the discussion with John and Wilbur after class. In that discussion Wilbur said he felt a little irritated, because Karin had said something that had nothing to do with what he was talking about at the time. He also told John and me that Karin was in his opinion one of the weaker students and therefore assumed that she just had been a little slow in following his reasoning. After some discussion he said he was sorry not to have realized that in fact Karin showed a deeper understanding of the commutative rule than just a formal acceptance.

Karin showed that she was really amazed. For many years as a teacher and in my discussions with my student teachers I felt that it is almost impossible to teach young children the commutative and the distributive rule as important mathematical properties. For most of them they are natural phenomena and I feel that making a big fuss about it is pretentious and only shows that mathematics is about making simple things difficult. Only when we are more sophisticated in understanding what mathematics is about, we learn that formally mathematics has to be built up from scratch and to do so we have to formulate axioms. These axioms are not just

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concoctions that somebody dreams up. They are based on practical experiences. In other words: the commutative and the distributive rules are not, as Wilbur and I and many other teachers had been trying to teach for a long time, *explanations* of phenomena, but indeed *formalizations* of such phenomena. I think that this is just what Karin felt: Karin's amazement was not that you can explain the phenomenon 2 _ 3 = 3 _ 2 with help of the rule, but that you can explain the rule with help of the phenomenon.

I am pretty sure that Karin did not realize this, but for me, as a teacher I learned from it that it pays to present situations to students in which they can get amazed as a starting point for learning to generalize. If Wilbur had realized this during the lesson he could have used Karin's remark in this way.

The same language, different meanings

There are several approached to reflect on this episode. I have chosen one that I found in an article (Klaassen & Lijnse, 1996) of two ex-colleagues of mine. They describe an episode in which a teacher did not succeed to convince a student. They give five different viewpoints from which one can analyze the episode. First, the view that an error was made and thus the student has to learn what was wrong and to repair it. Second, the view that there is a misconception and the teacher should work on that to help the student to get the correct conception. Third, the view that the idea of misconception is not an acceptable way of analyzing. Better is to recognize student's previous learning and talk about pre-conception. It is the teacher's job to find ways to confront that pre-conception and the 'correct' conception so that the student can change. Fourth, the view that the student and the teacher are living in a different world. "In each world, different concepts are used, being part of different kinds of knowledge, with different characteristics and problem-solving procedures" (ibid. p. 121).
The fifth view comes from the authors themselves. They argue that teacher and student do not live in different worlds. In fact they agree with each other, at least as one looks at the language they use from the outside. However they use the same language in a different way, giving the words a different meaning.

Interestingly in our case there was no mistake, so the first three viewpoints are not relevant here. We even cannot think here in terms of different worlds. The last view, however is enlightening.
We have to be clear that in our case the issue was not something about the commutative rule. In fact this rule was not relevant at all; it was just a means to get to the more general concept that certain kinds of rules are formalizations of certain phenomena. Following the line of Klaassen and Lijnse we can say that the language spoken by Karin and Wilbur was the same: Both talked about the fact that 2 time 3 is equal to 3 times 2, that this was so for every pair of numbers which could be expressed by $a \times b = b \times a$. They agreed about that, but each of them gave a different meaning to it. Wilbur did not recognize that. He thought that Karin was just a bit slow

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in taking up his ideas and therefore in a friendly way dismissed her remark as not relevant any more.

Reflecting on how I learned and its implications

From Karin I learned to distinguish between formalization of phenomena and explanation or a rule. In both cases one can use the same examples but in different contexts: in the context of formalization the examples are paradigms, in the context of explanation they are instances (Freudenthal 1978, pp. 201; Van Dormolen 1986).

Reflecting on my learning experience I found the same-language-different-meaning approach highly illuminating and useful in analyzing other episodes in which there is some sort of teacher-student conflict.

What I learned is interesting and illuminating. I used that later many times in my own classes of pre- and in-service teachers. More general is, however, the question of how I learned. Why was it that I could learn from Karin, while Wilbur and John did recognize her intentions only after I brought the matter up in our after-lesson discussion?

One essential element is (as ever so often) the role of the context. For me the situation was open, I had no restraints for attaining goals, like Wilbur. I was not on edge like John who had to cope with many new experiences. I had no responsibility like Wilbur to keep as many students as possible attentive and motivated. I could start to think about Karen's remarks without paying attention what was happening after that. Wilbur's context generated for him a situation that made him, experienced teacher as he was, react automatically. His experience with Karen's not so bright performances in the past made him - without realizing it - assume that her remarks would again be on the poor side. In the after-lesson discussion he realized and deplored this automatic-pilot-attitude and he had every intention to be more attentive in the future to unexpected interventions of his students.

There is more to say about context. It is noteworthy that in the descriptions of all three episodes, Groves, Zan and I apparently wanted to tell much more than – on first sight – seems necessary to describe the case. After writing down the first draft of my episode, I wondered if I had written too much. Could I not delete, for example, all what I wrote about John? On first sight the presence of John seemed to be irrelevant. Yet I could not bring myself to take him out of the story. Realizing this I found that, intent as I was on finding subjects to discuss in the after-lesson talk, I might not have noticed the importance of Karin's intervention without John's presence. I see similar situations in the episodes of Groves and Zan. Apparently the total context was crucial for the three of us as an opportunity to learn.

Last remark: All this came from reflection (Van Dormolen 1998, 2000), both in the classroom, in the after-lesson discussion and in the composition of these notes for the panel discussion. Crucial as elements such as context, attentiveness, open mindedness, teaching experience etc. are for learning, without conscious reflection I would not have learned at all.

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COMMENTS ON JOOP'S EPISODE - ROSETTA ZAN

The episode described by Joop can be read and generalised both by centring on the various actors present, and also by viewing it from a different perspective: one possibility, suggested by Joop, is to focus on the epistemological positions (either conscious or unconscious) of the teacher and the researcher. My reading of the episode comes from a different perspective, which is also the one that comes most naturally to me given my area of research: I will focus my attention on the complexity linked to the presence of two different subjects, pupil and teacher, and to their communication.

I tried to 'put myself in Karin's shoes', and I'll now describe her possible thinking process using a *virtual monologue* (Leron & Hazzan, 1997): this method, which originated from a researcher's attempt to better see the world with the student's eyes, provides us with an instrument for communicating *our own* way of seeing the pupil's point of view to others. When Wilbur starts to introduce the distributive rule, I am still following the previous topic about the commutative rule:

> It's not like before, when maths was simply about doing calculations. No,... now we're not just working with numbers to get a result ... Wilbur is teaching us something important, something 'grown up': we can describe things that seem simple in a more important way, it's not enough any more just to do a calculation. Something that seemed so natural... that 2x3 and 3x2 give the same result ...Wilbur explained to us that this is a property that has a name! It's important to be precise like this in mathematics: as we go on, we've got to learn to be even more precise! ...But... if we really want to be exact (and that's what we're learning to do, isn't it?,) ... who's telling me that 2+2+2 and 3+3 give the same number? Without calculating as we did before! Let's see... If we write 2+2+2 as 1+1..+..1+1...+...1+1, ...yes, that's it! and we write 3+3 as 1+1+1...+...1+1+1... there we are! Now, things are more precise!! I'm going to tell Wilbur... (and she puts up her hand)

Karin's effort reminds me of many mathematics students that are learning to prove some results by starting from the Peano axioms and the definitions of the + and the x operations. It is the effort to distinguish between what can be assumed as true and what has to be proved when we are moving in an environment where everything already seems familiar. Even if this effort is not always successful, and the student's solving processes highlight his confusion, it still testifies that the student is starting to approach mathematics in a new way: a more sophisticated way, linked to an evolving vision of the discipline, moving towards more mature epistemological forms.

Karin's intervention was a missed opportunity for Wilbur, but Wilbur had *his own* project in mind: 'to explain' the commutative and distributive rules. This project is the result of a decision made before he started his interaction with the pupils. There are many of similar decisions to be taken: how to present the topic, what examples to make, how much time to dedicate to the various steps.... As time goes on, such decisions get easier and easier: after all, Wilbur has been teaching for many years, and this is a well defined task for him! But once he starts interacting with the class, Wilbur

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finds himself having to manage situations that he hadn't completely planned for when preparing the project. He has to deal with the pupils' questions, and he has just a few seconds to decide *how* and *whether* to answer. Since the teacher's decisions are not just influenced by his knowledge, but also by his beliefs and emotions, this influence is particularly strong when there is little time available for deciding what to do. While Wilbur is explaining the distributive rule, Karin intervenes. Wilbur, who is so focused on his attempt to make the students understand distribution, is ready to answer questions or deal with objections on *that* property. But Karin's intervention moves away from the direction he wants to follow and puts him in difficulty: 'Yes, you are right, *but we saw that already*...'

It becomes natural to ask ourselves: what idea was Karin constructing? And how could her teacher have discovered it? What consequences will Wilbur's reaction have had on these new ideas? What consequences will this have had on Karin's desire to understand what was new in this activity? Will Karin try again to 'make sense' of mathematics? Or will she simply conform to the demands and pace of Wilbur, to the demands and pace of the lesson, to the demands and pace of others?

But this is obviously a general problem: how can we understand what sense of mathematics the pupils are making for themselves? How many Karins do we distance from mathematics because we do not know how to listen to them? What consequences does a rigid and limited schedule have on the pupils? What idea of mathematics will be constructed by pupils who haven't had an answer to their questions or haven't been able to ask their questions? And what will be the resulting image that they construct of themselves?

What about Wilbur? Again, this is a general problem: how can the teacher learn to make quick decisions? How can he recognise the consequences that these decisions have had? How can he correct his mistakes if required? And how can he learn to respect the time needed by the students, and to respect their thinking processes?

COMMENTS ON JOOP'S EPISODE – SUSIE GROVES

In one sense it is difficult to go beyond Joop's own analysis of this episode — in particular the discussion of the difference between *explanations* and *formalisations* of phenomena. However, for me, the most striking comment in Joop's description is where he says that he learned that "it pays to present situations to students in which they can get amazed as a starting point to generalise".

There are clear analogies with this view and the notion of creating cognitive conflict as a teaching strategy in science education. Vygotsky (1962) characterised children's *scientific conceptions* as developing downwards, while their *spontaneous conceptions* develop upwards, stressing the importance of the interaction between these, with spontaneous concepts enriching scientific concepts with meaning and scientific concepts offering generality to the spontaneous concepts. This is sometimes described as the vine metaphor. Pines and West (1985) adopted the vine metaphor as a

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framework, distinguishing different prototypes of learning situations by the extent to which the "upward and downward growing vines" clash or are congruent. From a constructivist viewpoint, they considered meaningful learning to take place when the two vines become intertwined, with the new scientific knowledge serving to make sense of the learner's world of experience. In many ways this appears to describe what was happening when Karen "intertwined" the formal statement of the commutative property with her observation that two times three and three times two "have the same number of one's".

While Wilbur, the teacher in Joop's episode, failed to capitalise on this opportunity — or even recognise it at the time — this is not surprising as he had a whole class to contend with and had already moved on to look at the distributive property. Moreover, if we are to take on board the notion of using students' 'amazement' as starting points for generalising, we not only need to find appropriate situations with the potential to amaze students, we need to acknowledge the idiosyncratic nature of such happenings. For example, who could have predicted Karen's reaction to a standard treatment of the commutative property, which presumably she had met many times before?

There are also implications for the organisation of teaching situations if teachers like Wilbur are to be able to capitalise on spontaneous situations such as the one described in this episode. In our *Calculators in Primary Mathematics* project — briefly described elsewhere in this paper — using the calculator, often in free play situations, prompted young children to be amazed. For example, one four year-old boy is captured on video spontaneously using his calculator to count by ones from a million. When asked whether he could count to one million one hundred he initially says he thinks there is no such number. However as he nears it he begins to have doubts and finally when he gets to 1 000 002 he is truly amazed to see he has "gone right past it". The fact that the rest of the children in the class were absorbed in their own calculator explorations allowed the teacher to spend four uninterrupted minutes talking to this boy and his partner, during which time she was able to "learn" something about the boy's understanding of very large numbers, spontaneously prompt him to try a potentially amazing activity, and engage him in discussion related to the outcome. This would not be easy to parallel in a secondary classroom.

Perhaps the real lesson from Joop's episode however is that we are unlikely to learn from learners if we do not listen to them. A lot of time is spent in teacher education in teaching teachers to "teach" . Perhaps a lot more should be spent on teaching them to listen and learn from their students. In mathematics classes, this is particularly critical in terms of the type of communications we engage in with our students and the social and "socio-mathematical" norms (Yackel & Cobb, 1996) which operate.

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EPISODE II: MARCO AND ANNA - EACH FOLLOWING THEIR OWN PATH – ROSETTA ZAN

The episode

The event that I'm about to describe happened during a meeting with teachers. Another University had invited me to give a lecture on the difficulties in mathematics: the audience was mostly teachers from higher education. In order to get them involved, I decided to give some examples that would be within their experience and ask them to perform an interpretative analysis. My first case was the incorrect use of brackets, a very common area of difficulty. I described the behaviour of a student, Marco, who had to multiply x+1 by x+2. He wrote: $x + 1 \cdot (x+2)$, but he went on: $x + 1 \cdot (x+2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$

My intentions were: 1) To ask for their analysis of the case, start a discussion, etc. 2) If no other opinions had emerged, I would suggest a possible interpretation of this systematic error: maybe Marco interprets the presence of brackets as a sign that indicates the order of precedence for performing operations, but as a personal shorthand for his own use. He doesn't therefore see them as signs to be used to communicate with others, with fixed and shared rules, but as notes to be ignored once the exercise has been completed; 3) to underline that, following this interpretation, Marco's behaviour is completely consistent. Marco had possibly put brackets where he sensed they were needed to highlight a certain order, but didn't use them when he felt there was no need because the order to be followed was easily apparent. In this interpretation, the fact that he performed the operations in the "correct" order shows that *his* use of brackets was correct.

The example caused an immediate reaction with all the teachers present. There was a chorus of: 'It's true ! they do that!' 'Mine do too...' When everyone had calmed down, one teacher raised her hand and asked to make a point: 'It's true: they often do this. But I always take off two points: one because they made a mistake in not using the brackets, there ... [and indicates $x + 1 \cdot (x+2)$]; and the other point because they make a mistake in multiplying, here... [and she indicates the equal sign in the expression $x + 1 \cdot (x+2) = x^2 + 2x + x + 2$]'. The other teachers reacted noisily, many burst out laughing. The teacher who had spoken (let's call her Anna) realised she was being derided by her colleagues and she repeated heatedly: 'two mistakes: two points off. *I explain to them* that they have made two mistakes.' Her colleagues looked at me before reacting, trying to understand what I was thinking. In order not to put Anna on the spot, I asked her without irony: 'and in *this way* ... do the students understand? Don't they make any more mistakes using brackets?' She replied with resignation, but still convinced: 'No, they keep getting it wrong... but there are *two* mistakes.'

The incident finished there: I continued with my program.

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What did I learn from Anna's observations?

My first reaction to Anna's intervention was disconcert and disappointment, even if I hid this behind my question. I had been talking about the constructivist model of learning and everyone (including Anna) had been following me with attention; I had talked about the importance of understanding what is behind a mistake if you are to overcome it, and everyone (including Anna) had agreed ... and then *that* intervention! The other teachers were as shocked as I was: at least they seemed to have understood the message. Thinking back as I was on the train home that evening, even if my intervention had been useless to Anna, it had been of help to the others, so the balance of this exhausting day could be said to be positive. But even so, I couldn't get over Anna's intervention: remembering her expression, her conviction in defending her position, but also her dissatisfaction in recognising the uselessness of it! There was something important under all of this, something that I couldn't grasp. But what? When I thought back to my initial program in which I presented the case of Marco, what I wanted to do was to push the teachers towards 'putting themselves in Marco's shoes', because in this way, they could begin to see the interpretative hypothesis I was making: to decide how to react, it is first necessary to ask yourself '*Why* has the student made this mistake?'. But after Anna's observations, when I tried to put myself in the shoes of *her* Marco, I felt a total sense of estrangement: I had been corrected and penalised, though I hadn't even made a mistake! My attention had moved from *why* Marco had made the mistake to the fact that it wasn't possible for Marco to realise that he had made the mistake. But how could Marco modify his behaviour in a significant and definitive way, if he didn't recognise he was wrong, if the consequences of his error didn't lead to a failure? In that period the Pisa group was doing research about the evolution of attitude towards mathematics: we set an essay on mathematics ('Me and mathematics: my relationship with maths up to now") for students at all levels of school, and then we analysed the essays elaborated. It was a sentence of another Marco that came into my mind at this point: *'...I constantly dedicated a lot of time, and I really wanted to succeed. But seeing that everything I did was wrong, I became resigned and came to the conclusion that maths and me weren't meant for each other ...'*. Marco slowly distanced himself from a discipline that he couldn't control: maybe because the things he did 'well' were corrected and penalised?

Anna and Marco: I couldn't help but thinking of them 'together', as if there was just one Marco, the one with the brackets problem and who *then* went on to write the essay, and Anna his teacher. Together, but each one following *their own* path, and, in the end, both of them in difficulty, or rather -and Anna helped me understand this - *their diversity was the reason* why both of them were in difficulty: each had their own goals, their own criteria for evaluating whether they had been achieved, their own epistemology. But also Anna and I, in the end, had followed different paths: paths characterised by different goals, and different epistemologies. And in the end we didn't manage to find each other either.

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Analysis and comments

The sense of estrangement that we feel when we put ourselves in the position of a Marco whose teacher has just taken off two points highlights the possible rift that exists between the perception that the teacher and the student may have of the same situation. Such rift is particularly apparent in the case of Marco and Anna, not just because Marco's result is correct, but also because the fact that the result is correct makes his personal use of brackets 'correct' (in the way we hypothesised before). But this episode was important to me because it forced me to focus my attention on the consequences that are usually generated when two different subjects, the teacher and the student, have to recognise an error or failure, a preliminary recognition that must occur before the recovery operation can start. This focus allowed me to reinterpret significant episodes that at first appeared separate, in a new single coherent perspective.

In the case of Anna and Marco, the rift is so large that Marco doesn't recognise any of the errors that Anna identifies: in the end, Anna recognises incorrect solving processes, while Marco is probably convinced that he has given a correct answer with correct solving processes. But a similar rift can be observed in situations that at first seem very different to this case. For example, if the student sets himself the goal of finding the correct answer, and the error does not affect the outcome, the teacher's perception of failure cannot be shared by the student. But if the student doesn't realise his failure, or that *that* specific behaviour is causing the failure, how can he invest resources to change his behaviour? It is therefore not only important to ask ourselves *why* the student made a specific mistake before intervening, but also and in the first place, *whether* the student realises that his behaviour is at fault. But this is not the only point. The sense of estrangement that a student feels when he sees processes or products being corrected that he doesn't recognise as being wrong can have extremely negative results in the long term. The student begins to feel he has lost control over his success / failure, and mathematics appears ever more as a discipline in which the only person who has any control is the teacher. The belief that mathematics is an uncontrollable discipline has very important and negative consequences. On the one hand, the student is constantly searching for what the teacher 'wants'. On the other hand, negative emotional aspects appear such as anxiety, frustration, and even fear and panic, and in this way a fatalistic attitude takes over, with a consequential passive and resigned behaviour.

These considerations led me to change the perspective under which I approached the problem of learning difficulties in mathematics. The interpretation suggested by this episode allowed me to observe that some elements were common to other significant episodes that seemed to be completely separate problems up to then. It made me also reflect theoretically on the construction of a single coherent framework, which would provide instruments to observe and modify significant phenomena. A first partial and temporary result was the characterisation of difficulty, which also follows on from the comparison with other theoretical studies, and generates new problems and directions for research.

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Traditionally, the problem of recognising difficulties in mathematics relates to the recognition of errors. This doesn't mean that the difficulties are identified with the errors: many researchers (see Borasi, 1996) underline the positive role of errors in the learning process. But even so, errors remain a strong indicator of difficulty. The reflections expressed above convinced me that a new *explicit* distinction must be made between errors and difficulty. The diagnosis of difficulty based on the recognition of error is apparently linked to objective elements (in fact the error is weighed and evaluated), thus risking to hide the intrinsic subjectivity of the observation / intervention process, caused by the presence of two different actors, the teacher and the student. Even if the teacher recognises the student's error and intervenes, it is up to the student to modify his behaviour: but if the student is to significantly change his behaviour he first has to be convinced that a change has to be made, that the existing behaviour leads to failure. Talking about a failed process is different to talking about mistakes: it means making reference to a goal that has not been achieved. Therefore it is only by setting things up in terms of goals, rather than as 'objective' errors, that we can highlight the presence of two different subjects, student and teacher, and the complexity that derives from this. In the same way, we can consider managing this complexity only if we recognise explicitly that it exists.

All these considerations led me to conclude that it is better not to identify errors as indicators of difficulty: we should instead concentrate on failure, intended as not having reached the desired goal. The reference to goals takes us back to points underlined in some studies (Cobb, 1986): the problems that students face in the context of learning mathematics are 'more social than mathematical'. We have to therefore consider the possibility that the goal set by the student is not necessarily 'mathematical': it can happen that when setting an exercise, the teacher sets a goal (e. g. find the area of a certain figure) which provokes a different goal in the student (e. g. give the right answer to the teacher). We should also consider that the points made here regarding the didactical value of errors suggest that only *repeated* failures and not a single one should be considered as a difficulty indicator. These observations bring me to the following temporary working definition of difficulty: the repeated failure in problems that the pupil encounters in the context of mathematics education. The effectiveness of this definition should be measured in practice, particularly in terms of its capacity to provide new instruments to detect and modify significant phenomena, opening up new directions for research.

LANGUAGE AND CONTEXT: COMMENTS ON ROSETTA'S EPISODE – JOOP VAN DORMOLEN

Anna and Marco

I would like to comment on the episode presented by Rosetta from the background of same-language-different-meaning approach that I wrote about in my comment on my own episode. Both used the same language, in this case the sequence:

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Multiply $x + 1$ with $x + 2$

$x + 1 \cdot (x + 2) = x^2 + 2x + x + 2 = x^2 + 3x + 2$

Both were using the same language but gave it different meaning: Marco considered it right, Anna said it was wrong. Rosetta did not write about this in these terms, but her remark that Anna and Marco followed different paths seems to agree in essence with the same-language-different-meaning approach. I would like to put forward that the language approach may give us a more promising possibility to get Anna and Marco to come to terms with each other. The different-paths metaphor makes us a little hopeless. How can they ever come together? That is the danger with metaphors: They make you understand, but they do not show how to go on. The language approach is not metaphoric at all. It describes real life situations. As such it helps us not only to understand the situation; it also might give us an indication how to repair the discord between the two speakers. How can we get them to speak the same language with the same meaning?

For that we have to analyze the situation a little more. That was what Rosetta had been doing. She wanted the teachers to try and understand "what is behind a mistake...". Yet she did not convince Anna. Apparently Anna agreed as in general terms, but when she imagined that something like the example would happen in her class, she became uneasy and had the courage to give her opinion. What made her unhappy with the Marco example?

I would like to propose two reasons. One, as Rosetta points out, is the inability to put herself in Marco's shoes. The other is the divergence between Anna and Rosetta.

Let me try and analyze the first reason first. I learned from Karen (see my own episode) that thinking in terms of analyzing mistakes is an asymmetric way of working. It is the I-know-better attitude. The same-language-different-meaning approach makes one realize that there are two sides in a conflict. Both sides are right when viewed from each owns standpoint. So let us try and find out what the different standpoints are.

Again I see a similar situation as in the Karen-Wilbur conflict that I described in my own episode. Marco's path is *formalisation* of an experience, while Anna wanted to *explain* the procedure from the rules. Seen from this standpoint both are right in their conclusion: Marco thought he did the right thing, namely reply to the instruction to multiply x + 1 with x + 2 and for that he invented an ad-hoc formalisation, that helped him to find the solution, but is irrelevant for others. From his viewpoint his teacher had no business with the way he worked. He gave the correct answer to the task. Anna thought to explain what he did in terms of allowable rules and as such –from her viewpoint- he went wrong.

Anna and Rosetta

Much more important for the case than the Anna-Marco conflict (they even did not know each other) is the Rosetta-Anna conflict. Analyzing it with the same-language-

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different-meaning approach one could say that Rosetta and Anna were using the same language when talking about what Marco did, but Rosetta (and the other teachers) were thinking about an explanation of his behaviour, while Anna was thinking in a for her more direct practical direction: What to do about it? At that moment she could not find another solution but the one she was used to. Putting myself in Anna's shoes I would say something like: "All this talk is good and interesting, but it does not tell me how I can make Marco aware of what he has done and why I cannot accept that".

Rosetta, in her reflections, takes a wider approach, which brought me to think about the different contexts in which Rosetta had put herself (and most of the teachers with her) and Anna's context.

Rosetta's context was the constructivist model of learning. As long as this went in general terms it was also Anna's context, but the example of Marco changed that. Just as the other teachers she recognized the situation from her own experiences. Maybe because of that, the situation became more realistic for her than for the others and she wondered what she had to do when she had had Marco in her class. She could not understandingly just accept Marco's work. She knew she had to do something. She asked herself how she had to react. That changed her context completely. She had to make Marco aware that this is not done and therefore she had to punish him.

Anna's context made her say things that – in her fellow teacher's eyes – transformed her in the stereotype of the old fashioned teacher that wants her student just do what they are told. In a way this is similar to Wilbur's generalization of Karin's abilities in my episode. (From reading Rosetta's analysis I am sure that this was not how she saw Anna!). Anna apparently was not aware of her other context and therefore expressed herself rather crude and cruel, but maybe what she said was a clumsy way of asking for help: "Tell me what one can do in such a situation". Was this a (rather extreme) instance of same-language-different-meaning?

What did I learn from Rosetta?

I learned nothing new, but am grateful for her description and her reflection on it, because I saw the same elements as in my and Susie's episode:

- The reason of a discord can often be analyzed with the same-language-different-meaning approach. It makes the situation symmetric. There is no I-know-best situation any more.

- In mathematics teaching-learning such different meanings can sometimes (often?) be explained as difference between formalizing experiences and explaining rules.

- The same-language-different-meaning approach may also prevent us from generalizing other people's abilities.

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COMMENTS ON ROSETTA'S EPISODE - SUSIE GROVES

For me the most striking observation about the episode presented by Rosetta is the critical importance of people's beliefs about the nature of mathematics for their behaviour as learners and as teachers of mathematics.

The first of the two Marco's referred to in the episode appears to (not uncommonly) view mathematics as a set of procedures to be mastered in order to obtain correct answers to problems which of themselves appear to have no meaning and over which they have no ownership. Having obtained the correct answer of $x^2 + 3x + 2$ for the product of $x + 1$ and $x + 2$, Marco presumably was not concerned by the fact that his use of notation in the process of obtaining the answer was unorthodox. In my experience, students when asked what they mean when presenting a written or verbal solution often answer "You know". When pressed further they still repeat "You know" as often as needed. This is not surprising because usually the teacher DOES know and is only asking for confirmation from the students that they too "know" whatever it is the teacher knows. Students have very little experience of being asked to solve problems where it is possible that the teacher does NOT know the solution strategy or thinking used by the students — that is situations where they genuinely need to explain their thinking to someone else. They have little idea that mathematics needs to be communicated to other people, both within the community of mathematicians and when applying it to problems in the real world — and of course that this is why mathematical conventions and their correct use are important.

By way of contrast, Anna in this episode has a very strong sense that mathematical conventions are important. But there is no sense that she sees these as important because of the need to communicate mathematics to others. In fact, she appears to share Marco's view of mathematics as a possibly meaningless, but nevertheless rigid, set of rules and procedures which it is her task to transmit to her students, however unsuccessfully. A constructivist model of learning *mathematics* is by and large irreconcilable with such a view of mathematics, so it is not surprising that both she and Rosetta felt confronted by the mismatch between Anna's response to the Marco scenario and her apparent acceptance of the constructivist model. While I realise that constructivism refers to how knowledge is constructed by humans, regardless of the context or the discipline, I have italicised the word *mathematics* in the previous sentence because I wonder what we believe might be the implications of adopting a constructivist model of learning for teaching someone, for example, how to fly an aeroplane — which I am assuming here is strictly a set of skills. Would holding Anna's view of the nature of mathematics simultaneously with a constructivist model for the learning of mathematics lead to an impossible notion somewhat akin to "discovery learning"?

The second Marco in Rosetta's episode presents a sad story of really trying to succeed, but seeing "everything I did was wrong" and coming to the conclusion that he and maths "weren't meant for each other". This story appears to illustrate that this Marco also saw mathematics as a set of rules and procedures which need to be learned

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verbatim and mimicked in response to an appropriate cue. Again there is no sense of mathematics as a living, evolving discipline, with intrinsic underlying meanings which can be explored and personally constructed into a web of knowledge.

In her description of the episode, Rosetta, while seeing Anna and the two Marco's together, sees them all following their own paths, with their own goals and criteria. In my reading of the episode, I was much more struck by the similarities of their goals and criteria and their common, mechanistic view of the nature of mathematics.

EPISODE III: CALCULATORS – THE FIRST DAY - SUSIE GROVES

The episode

The event I have chosen to describe occurred ten years ago when Jill Cheeseman and I were producing the video *Young Children Using Calculators* (Groves & Cheeseman, 1993) to disseminate some of the findings from the *Calculators in Primary Mathematics* project.

The context

The *Calculators in Primary Mathematics* project was a long-term investigation into the effects of calculator use on the learning and teaching of primary mathematics. Kindergarten and grade 1 children in six schools in 1990 were given their own calculator to use whenever they wished in class. The project followed these children through to grade 3 and 4 in 1993, with new children joining the project each year as they started school. The project was based on the premise that calculators, as well as acting as computational tools, are highly versatile teaching aids which can provide a mathematically rich environment for children to explore. Teachers were not provided with classroom activities or a program to follow, instead they were regarded as part of the research team investigating the ways in which calculators could be used in their mathematics classes.

At the time when this event occurred, video footage had been obtained in a number of classrooms and we were ready to edit it to produce the final videotape. However, we were alerted to the fact that we had missed an important aspect when a teacher at one of our regular teacher meetings showed her "home video" of children in her kindergarten class being given their calculators on the first day of school. For technical reasons we could not include her video footage and it was too late to capture the first day of school in any of the project classrooms. However, for reasons which no one could remember, one of the six schools had decided that the project would commence at grade 1 rather than kindergarten. We approached the school and asked whether they would be prepared to include their kindergarten class in the project and allow us to videotape the lesson in which the children would be given their calculators. The school and the kindergarten teacher, who we will call Clare, agreed.

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The event

A week before the videotaping was to occur, we visited the school with the technical producer to discuss what would happen in the lesson, space constraints, etc. For reasons of space it was decided that the class would be split into two and each have a 20 minute session. Unlike the other lessons videotaped (and all other lessons taught in the project) where we left it entirely up to the teachers to plan their lessons, on this occasion we particularly wanted to capture footage similar to what we had seen on the "home video" and in many other project classrooms over three years — i.e. children being given their calculators and asked to explore them and then report back to the class what they had found. Clare, who at that stage had had no previous connection with the project, readily agreed to the proposed plan.

Just before we left, the producer asked me to recap with Clare exactly what would be happening the following week. Clare thought for a while and replied that as it was a Thursday, she would be dealing with number and that as it would be the ninth week of the school year, she would be dealing with the number nine. The children would be doing some colouring and other activities based on the number nine and five or ten minutes before the end of the lesson she would hand out the calculators and ask the children to enter some nines on it. This was not what we had expected and we asked her whether she would consider devoting the entire 20 minutes to the plan we believed had previously been agreed. This time she agreed somewhat reluctantly, saying that she did not believe that children would be productively engaged for such a long period.

During the two 20 minute sessions, extracts of which appear on the final videotape, children excitedly explored what they could do with their calculators, while Clare engaged them in purposeful discussion about what they were doing. One boy opened his calculator and exclaimed: "Ooh, now I can tell the time!" One child entered the numbers 12345678 and was disappointed when he had to clear the display in order to continue with 9101112. Another child showed the teacher a display which included a negative sign and then demonstrated, at Clare's prompting, how to "take the sign away". A girl entered 98765432 and answered Clare's question regarding the significance of the numbers by saying that "that's when a rocket ship blasts off". Most remarkable was the child who entered 92430 and explained that this was the date — the teacher had recorded the date on the board for the first time that morning. But of course it was recorded as 30/4/92.

What we learned

The most striking thing that I learned was the fact that I could not take for granted that each of the parties in a discussion took away the same understanding of what had been discussed — on my part, I "took as shared" the understanding that the children would freely explore their calculators and then report back to the class, while Clare framed the discussion in terms of her previous understanding of what a lesson at this stage of the year would look like.

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More importantly, I also learned what an excellent teacher Clare was. Although unwilling at first to follow the proposed plan for the lesson, she did so with enthusiasm on the day and her questioning of the children both during the exploration time and the sharing time resulted in high quality discussion. She was very good at listening to the children and asking probing questions in order to understand the purpose of their often surprising uses of the calculator.

Clare also learned what a wide range of number knowledge was present in her class and how by lock-stepping the whole class into the same, relatively low-level activity, she was only catering for a very small minority of the children. Not only did Clare acknowledge this, but the role of the calculator in revealing this common feature was reported frequently by many teachers throughout the project's duration.

Both Clare and I also learned from the children new ways in which the calculator could be used not just as a computational tool but also as a tool to facilitate mathematical exploration and reflection. In fact, in much the same way as illustrated here, the project had already found that very young children frequently used the calculator as a "scratch pad" to quickly and easily record numbers and number patterns which they found particularly significant. None of us had anticipated this use of the calculator. Once teachers noticed this spontaneous use, many devised innovative learning activities based on using the calculator as a recording device.

Students as learners and teacher professional growth

Since Clare joined the *Calculators in Primary Mathematics* project at the stage when kindergarten classes only continued to be part of the project for a little over six months and she was not one of the seven teachers who were part of our case study investigating the effects of calculator use on classroom practice, I am unable to comment further on what happened to Clare after the event reported here. However, from the wealth of data collected in the project, I believe that Clare's experience was shared in many ways by many of the teachers in the project. In particular, I chose this event because in my opinion it highlights the way in which teachers' learning from their students' learning experiences can play a powerful role in their professional growth. I will try to elaborate on this below.

The two major aims of the project were to investigate the effects of children being given 'their own' calculators to use freely from kindergarten onwards on firstly the classroom practice of teachers and secondly the long-term learning of children. While funding for these two 'investigations' came from separate sources, they were nevertheless seen as inextricably linked from the outset. In particular, an underlying assumption of the project was the belief that teachers would observe children using their calculators – presumably in ways which challenged their existing beliefs about the nature of young children's learning of mathematics – and that reflection on the 'classroom experiment' of calculator use and the consequent observations of children would lead to a change in their teaching practice.

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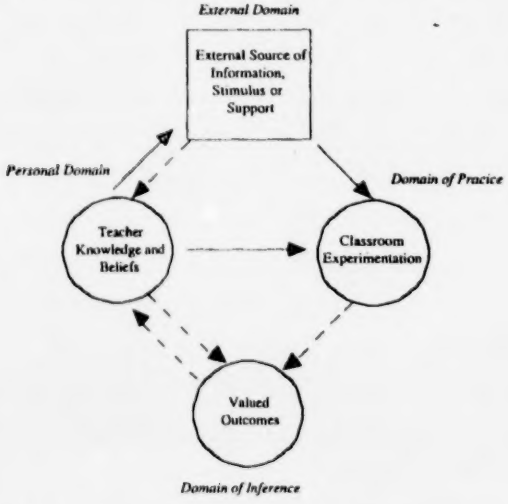
This assumption was originally based on Guskey's (1986) model of teacher professional growth, which proposed that the major motivation for teachers to change is the desire for improvement in student learning outcomes and that changes in teachers' classroom practice need to precede changes in their beliefs and attitudes. Thus teachers learning from children's learning was a central thrust of the project from the beginning.



Key: Solid line = *enactive* mediating process; broken line = *reflective* mediating process

Figure 1. **The Clarke-Peter model of professional growth**

Later in the project, we adopted Clarke and Peter's (1993) dynamic model of teacher professional growth, which traces its origins to Guskey's model. The Clarke-Peter model (see Figure 1) identifies four domains: the *personal domain* (teacher knowledge and beliefs); the *domain of practice* (classroom experimentation); the *domain of inference* (valued outcomes); and the *external domain* (sources of information, stimulus or support). It further identifies *reflection* and *enaction* as two mediating processes which are used to explain how growth in one domain is translated to another. In recognition of its dynamic nature, the model allows for entry at any point in the cycle.

In terms of the Clarke-Peter model, the stimulus of the presence of the calculator, together with the support provided by the project, were changes in the external

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domain. This was translated into action in the domain of practice through classroom experimentation, which, in this case, took the form of using calculators on a regular basis. As well as their own observations of children learning in the calculator environment, teachers received feedback from the members of the project team who reported their observations of individual children's learning during regular classroom visits. These visits therefore acted as a further stimulus by giving teachers access to a much wider range of observations of children than would otherwise have been possible. This in turn provided teachers with enhanced opportunities to engage in the reflective process in order to bring about change in the domain of inference — that is the valued outcomes. The reflective process was also supported by the discussion which took place at project meetings. This reflection on changes in both the external domain and the valued outcomes mediated change in the personal domain.

Most of the seven teachers who were part of the case study investigating effects of calculator use on classroom practice claimed to have made substantial changes to their teaching of mathematics, with all seven commenting that their mathematics teaching had become more open-ended, and four teachers describing their mathematics teaching as having become more like their teaching of language (Groves, 1993). While it is not possible to determine the long-term effect on Clare of the event reported here, it appears highly likely that Clare's teaching also became more open-ended as a result of her experiences that day and also when subsequently working with calculators with children in her class. Certainly the findings from the project in general support the notion that placing teachers in experimental situations which challenge their existing beliefs by focusing attention on their students' learning has the potential to support teachers' own learning and professional growth.

COMMENTS ON SUSIE'S EPISODE – ROSETTA ZAN

Reading this episode gives us the possibility of reflecting on some significant aspects of the teaching process. Even though the information that can be extracted from a single episode are limited, Clare's behaviour reminds me of behaviour and decisional processes observed in other teachers, and of possible interpretations of these processes.

In a first instance Clare refuses the work proposal of Susie and Jill: she thinks that the children cannot be left for 20 minutes to perform that type of activity, and that a more structured activity under the teacher supervision would be better. I seem to recognise in Clare's behaviour, but above all in her decisions, the influence of her beliefs and emotions. Her beliefs regard both her class and their capacity (the children cannot be left working for a long time on a single activity), and the teaching of mathematics (if we want the children to learn, we have to provide them with structured activities). But I also recognise a strong emotional component in all of this: the fear of losing control of the situation, and the anxiety of not completing a pre-established program.

The importance of the theme of teacher's beliefs in the research on teaching is linked to the shift of this research from the only observable phenomena such as teacher's

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behaviour, to studies about the teacher's decision processes. In the more recent studies teachers are seen as thoughtful professionals, who make judgements and carry out decisions in a complex environment. The teacher's decisions are influenced (just like the decisions made by a subject involved in a problem posing or solving activity) both by their knowledge and by their beliefs. In particular, the beliefs regarding the capacity of their pupils can push the teachers towards making low demands, and can end up in low results, as suggested in pioneering studies by Rosenthal and Jacobson (1968) on the so called Pygmalion effect. However, research on problem solving highlights that decisions made by a subject who is solving a problem are also influenced by his/her emotions (Mc Leod & Adams, 1989): therefore the teachers' emotions, which are currently studied less than their beliefs, have just as important a role in explaining their decisional processes (Malara & Zan, 2002).

These factors (beliefs and emotions) also have a great influence on the relationship between teachers and researchers, and can be an obstacle to the implementation of the researchers' project. Even if Clare declares, and is convinced that she wants to participate in the project, her co-operation is held back by her resistance to allowing the children to work freely for a longer period of time. This phenomenon has also been analysed in mathematics education with respect to the reproducibility of a teaching experiment (Arsac et al., 1992): even when the teachers' behaviour has been programmed down to the finest detail, some of their non programmed decisions at a micro level are enough to make the activity not reproducible.

Reflection on this type of episode can lead both the researcher and the teacher to modify their practices. As a teacher these reflections convinced me of the importance of being aware of my decision-making processes, and of the deeper motives that bring them about. As a researcher, reflection on the role of the teacher convinced me that the reproducibility of a teaching experiment depends on the training given to the teachers involved in the experiment. Such training cannot simply be limited to communicating the spirit of the project to the teachers, but has to try to work not only on their knowledge but also on their beliefs and their emotions.

Analysis of episodes like this one also gives the opportunity to reflect on the importance of allowing pupils to explore, and not to limit their mathematical experience to very structured tasks. If problem solving is a crucial activity in the development of pupils' abilities and knowledge, the task of problem posing is no less so (Silver, 1993). In particular, the fact that during problem posing the subject solving the problem is the same as the one proposing it, has important implications. It allows certain cognitive obstacles to be overcome, such as those linked to the comprehension of a text, which are often exasperated by the stereotypical nature of school problems. As well as this, since exploration stimulates the production of conjectures, problem posing provides an ideal context for constructing the first steps in proving processes that are crucial to mathematical activity. But above all, problem posing guarantees active involvement of the pupil, reducing the risk that mathematical activity loses its sense and that the pupil loses control, thus generating a negative and fatalistic attitude.

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Concluding remarks

What I presented here is certainly only one of many possible interpretations of Clare, Karin and Wilbur's behaviours, but it is also the result of other experiences and analyses performed on other pupils and teachers. What I want to underline is the plausibility of this interpretation and NOT its correctness! Such plausibility is sufficient to highlight that the lack of communication between teacher and pupil (that I see as common to the three episodes) can be or become cause of difficulties. But I think that highlighting the complexity linked to these aspects takes us towards having instruments to make it manageable. The pupil/teacher relationship is asymmetrical: in my opinion managing to listen to and observe the pupils, talk to them, monitor the vision of both mathematics and themselves that they are building up, are all components of the teacher's role. This raises new issues which researchers will have to face: the need of developing a theoretical framework providing teachers and researchers with tools for observing and interpreting pupils' behaviour, and the need of adequate teacher education, in order to develop teachers' knowledge, but also their sensitivity and awareness (Mason, 1998). Theoretical tools, sensitivity and awareness can help the teacher recognise that the pupil is following a different path from the one he or she was expected to follow, and consequently make adequate decisions to reduce the distance between reality and expectations. But knowledge and metacognition are not enough without a crucial emotional aspect: the *desire* of communicating. Hence it is also on such desire that researchers need to work, in order to make teacher education act upon and affect such emotional factor.

Similarly, in my opinion, theoretical tools and metacognitive abilities are also the instruments that allow teachers and researchers to learn from learners: but, again, no tools are sufficient without the *desire* of learning.

WHAT I LEARNED FROM KARIN, ROSETTA AND SUSIE – COMMENTS ON SUSIE'S EPISODE – JOOP VAN DORMOLEN

I started with my own episode, and then I worked on the Rosetta Zan's. Susie Groves's episode is the last of the three on which I have to comment. I cannot but see several common elements in all three descriptions and therefore, next to commenting on her episode I use this, for me last, scribbling for a sort of summing up.

Same language, different meaning

The three episodes of Susie Groves, Rosetta Zan and me have several common elements. One of them is the phenomenon that people can talk with each other, using the same language and yet give different meaning to it. In Susie's story there is the agreement that "the class would be split into two and each have a 20 minute session." These 20 minutes meant for Susie that the whole period would be devoted to exploration of calculator, while for Clare the splitting up was meant just for technical reasons as there was not space enough for the camera crew in a full classroom. For the

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rest she though she could go on with her planned lesson, be it that five or ten minutes before the end would be devoted to the calculator. The intervention of the producer made it clear that there was indeed a difference in meaning. As ever so often this shows again the usefulness, if not importance, of conducting an evaluation after a meeting in which, between other issues, participants look back on their decisions: Are they clear for everybody? Does everybody attach the same meaning to it? (Johnson & Johnson 1997, Van Dormolen 2000).

The same-language-different-meaning is also clear in the actions and explorations of Clare's children. The language of the numbers in the display was indeed for them very different from the language in which the calculator is a computational tool.

Susie rightly points out Clare's willingness and ability to learn her children's language and to talk with them in that language in order to give them more and deeper experiences.

The context

Another element that is strongly apparent in all three episodes is the influence of the context on learning. All three of us found it necessary to go into detail about the circumstances that leaded to our learning. Each of us could have given the incident about which we wanted to write in less than half a page, but we decided not to do that. The context helped us to learn, so we were compelled to describe the context. For the children it was the context of being able to freely explore the new instrument. For Clare it was the context of being in the classroom with her children and noticing their excitement. For Susie it was the context of having the class as part of her project which gave her the opportunity to observe and learn.

Like ever so often the three episodes showed us again the importance of creating a rich context in order to provoke meaningful learning (Freudenthal 1978, pp. 178-185). The ideas of realistic mathematics education are based on this assumption (or should I say: law?).

Formalization and explanation

Explanation is about rules, formalization is about experiences. Both are elements in language and may help to determine if there is a case of same-language-different-meaning and if yes, to describe the different meanings.

From Susie's description I get the impression, that neither she, nor Clare wanted to explain rules about how to use the calculator. It is not clear what they expected what could happen if there was no explanation, but both of them were open minded to give children the freedom to find their own formalizations. One could ask if the word 'formalization' is a bit heavy in this case. I think not. Formalization is for me the result of some generalization. The children in Susie's episode, at least the children that she mentions, generalized and by telling Clare what they found, they formalized their findings.

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Never generalize on another person's abilities

In my episode Wilbur had certain experiences with Karin's abilities to learn mathematics. Unconsciously he generalized these experiences and that prevented him to recognize her intervention as constructive. In Rosetta's episode something similar happened to the teachers who seemingly stereotyped Anne as a rigid teacher. Wilbur's context was not a favourable one in which he could recognize his own bias. The same can be said about Anna's fellow teachers. Rosetta, Susie and Clare were in a different position. Their context was different, which allowed them to be aware that the situation should not be understood automatically. Clare was convinced that her children could not very well cope with the calculator and therefore did not want to spend more than 10 minutes to it. She was however open minded enough to try something else and, in contrast to Wilbur, she had all the time to make a decision to change her plan. Susie might have been shocked by Clare's intentions and did what she could to change Clare's mind. Yet she seemed to allow Clare to act as she looked fit. Susie's context was a very favourable one: an important goal of her project was just to be open and see what happens. Rosetta was shocked by Anna's remarks, did not allow herself to act accordingly. In this case she could not decide on the most favourable reaction. Her context restrained her in the time to find that reaction. Only later in the train she could reflect on the incident.

EPILOGUE – JOÃO FILIPE MATOS

When people find out that they have competing versions of ambiguous events, they often try to negotiate a kind of commonly agreed upon definition of the situation. This we can find in all three episodes just presented and analysed.

One of the features that seem to be equally relevant in the three episodes and subsequent analysis is that authors report learning within a certain situation reconstructing the conditions where it occurred and re-contextualising the learning within their actual frame. It is not surprising that all three authors do believe that learning occurred in the episodes shown and that they are able to identify the sources of learning in interaction with others.

There are also some aspects that emerge from the several analysis presented and that can help us to formulate relevant research questions regarding the issue of learning from learners.

Firstly, learning is seen as a phenomenon inherent to human nature and therefore an ongoing and integral part of everyday life. In fact, it doesn't seem possible to see learning as a special kind of activity separable from the rest of our life. This brings us to questions such as the definition of the unity of analysis in research on learning — focusing on people acting in the world *versus* focusing on cognitive processes.

Secondly, learning is conceptualized as an ability to negotiate new meanings, involving the whole person in dynamic participation in practices. Learning in general (and

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learning mathematics in particular) is viewed as a constructive activity that is developed more in a process of socialisation rather than one of instruction. It is during this process—which is socially mediated by language and other communication tools—that patterns of thought and action develop and are considered to be (mathematically) legitimate. This brings along for example the question of the contrast that is established between the concerns arising when we intend to teach children to *do* mathematics (emphasis on knowledge as a way of doing) and the concerns arising when we intend to educate them mathematically (highlighting a concern with a form of knowledge).

Thirdly, learning transforms one identity, by transforming one's ability to participate in the world and it does this by changing who we are, our practices and the communities we belong to. This very fact makes emergent issues such as the way the participant in the practice develops a sense of *belonging* and therefore construction of identity becomes a key element.

Finally, learning involves an interplay between the local and the global. The three episodes help us to realize how learning takes place in practice as it defines a global context for its own locality. Learning occurs whatever the educational form that helps to create the context for learning (including the absence of an intentional educational form) pointing to a fundamental distinction between learning and intentional instruction. Obviously this idea does not deny that learning may occur where there is teaching, but it does not make intentional teaching to be the source or the cause of learning.

All these features of learning need to have the attention of researchers in mathematics education. This paper aims to be a contribution for a first step towards an *ethnography of learning from learners.*

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